Shadowing is strong in tent maps

Mate Puljiz
joint with C. Good & P. Oprocha

University of Birmingham

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Joint work with…

Chris Good

Piotr Oprocha

A question

Question (Barwell, Davies and Good, 2011)

For an interval map, $f : [0,1] \to [0,1]$ with *shadowing*, is it true that a set $L \subseteq [0,1]$ is *internally chain transitive (ICT)* iff $L = \omega(x, f)$ for some $x \in [0,1]$.

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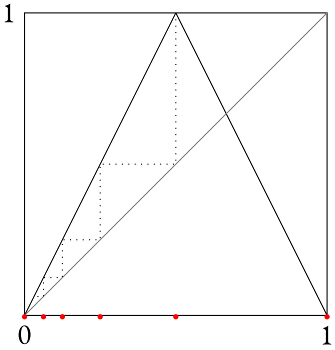
- A system (X, f) has the *shadowing property* if for every $\varepsilon > 0$ there is $\delta > 0$ s.t. every δ-pseudo orbit is ε-shadowed. (m. (X, f) is close to being a SFT)

Why should one care?



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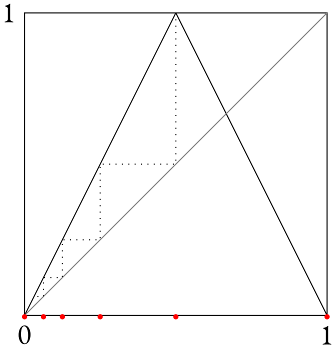

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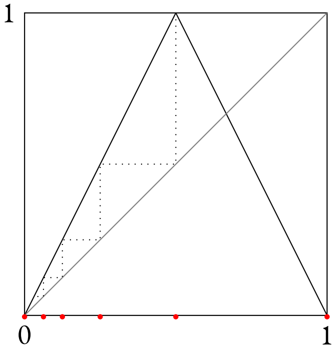

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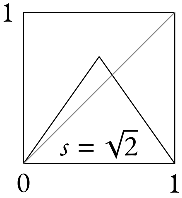
- 

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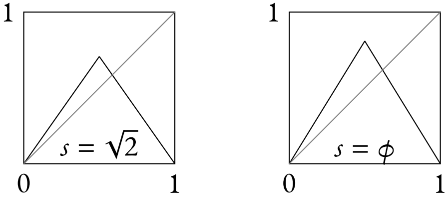
- 

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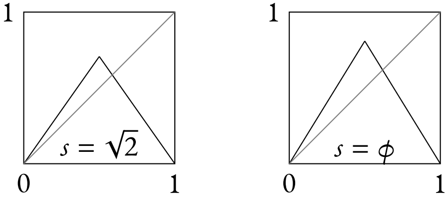
- 

- Tent map with slope s has shadowing for almost all $s \in [\sqrt{2}, 2]$, but the complement is dense (Coven, Kan and Yorke, 1988)

Another kind of shadowing

Definition (Limit (or asymptotic) shadowing)

We say that $f : X \to X$ has *limit shadowing* if every asymptotic pseudo-orbit is asymptotically shadowed.

We say that the sequence $\langle x_0, x_1, x_2, \dots \rangle$ is an *asymptotic pseudo-orbit* provided that $\lim_{i\to\infty} d(f(x_i), x_{i+1}) = 0$.

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Question

Shadowing $\overset{?}{\Longrightarrow}$ Limit shadowing $\Longrightarrow$ $\omega = ICT$?

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Idea of the proof:
(Chen, 1991) Shadowing for these maps holds m. iff

$$(\forall \varepsilon > 0)\ (\exists N = N(\varepsilon) > 0)\ (\forall x \in X)$$
$$B(f^n(x), s\varepsilon) \subseteq f^n(B_n(x, \varepsilon)), \qquad \text{for some } n \leq N.$$

where $B_n(x, \varepsilon) = \{y \in [0, 1] : |f^i(x) - f^i(y)| < \varepsilon \text{ for } i = 0, \ldots, n\}$.

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  $$\text{Shadowing} \Longrightarrow \overline{\omega} = ICT \Longrightarrow \omega = ICT$$

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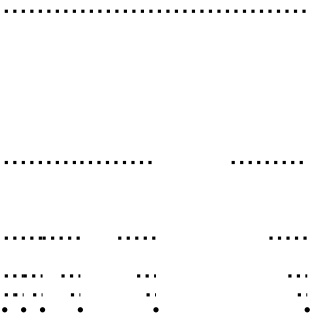
Question (still open)

Does every interval map $f : [0,1] \to [0,1]$ with *shadowing* also have the *limit shadowing property*?

A counterexample



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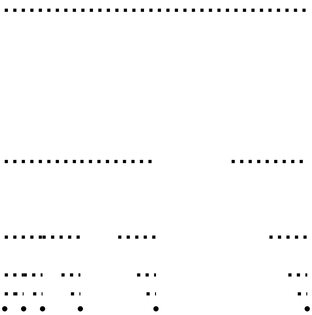

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- X_k is SFT with forbidden words $\{11, 101, \ldots, \underbrace{10\ldots01}_{k\text{-zeros}}\}$
- X_∞ is a subshift consisting of sequences with at most one 1
- This system has shadowing but not limit shadowing

Thank you for your attention!

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